

UNIVERSALLY SACKS-INDESTRUCTIBLE COMBINATORIAL FAMILIES OF REALS

V. FISCHER AND L. SCHEMBECKER

ABSTRACT. Given a maximal combinatorial family \mathcal{F} of reals, we say that a forcing \mathbb{P} preserves \mathcal{F} iff \mathbb{P} forces \mathcal{F} to remain maximal in the generic extension. We also say that \mathcal{F} is \mathbb{P} -indestructible. We prove that if a type of combinatorial family of reals is arithmetically definable, then every family of this type which is indestructible by the product of Sacks forcing \mathbb{S}^{\aleph_0} is in fact universally Sacks-indestructible, i.e. it is indestructible by any countably supported iteration or product of Sacks-forcing of any length.

In order to obtain this theorem, we prove the following result: For every condition $p \in \mathbb{S}^{\aleph_0}$ in the product of Sacks-forcing the statement “ p forces an arithmetical property of the generic sequence s_G ” can be translated into an equivalent Π_3^1 -statement.

We then verify that basically any type of combinatorial family of reals considered in combinatorial set theory (e.g. mad families, maximal cofinitary groups, maximal independent families, ultrafilters, unbounded families, splitting families, ...) can be defined arithmetically, so that our theorem applies. Finally, under CH we provide a unified construction of universally Sacks-indestructible witnesses for many different combinatorial families of reals - most notably for maximal cofinitary groups.

INSTITUTE OF MATHEMATICS, UNIVERSITY OF VIENNA, KOLINGASSE 14-16, 1090 VIENNA, AUSTRIA
Email address: vera.fischer@univie.ac.at

INSTITUTE OF MATHEMATICS, UNIVERSITY OF VIENNA, KOLINGASSE 14-16, 1090 VIENNA, AUSTRIA
Email address: lukas.schembecker@univie.ac.at